EXERCISES

1. Compare the ratios of the frequencies of transverse vibrations in the glockenspiel bar in Fig. 13.2 with the theoretical ratios for a thin rectangular bar given in Section 13.1. Can you account for the difference? (*Hint:* Compare Fig. 13.6.)
2. Write an expression for the frequency (f_1) of the lowest mode of vibration of a rectangular bar in terms of its length L and the speed of sound v_L.
3. Write an expression for the frequency ratio of the lowest transverse mode in a bar to the lowest longitudinal mode. Find this ratio for a glockenspiel bar 21.4 cm long and 0.90 cm thick ($K = t/3.46$). Compare this ratio with the ratio of the corresponding values given in Fig. 13.2.
4. What are the actual ratios of the numbers 81, 121, and 169 (discussed in Section 13.5)? How close are they to the ratios 2 : 3 : 4?
5. Using the formulas for longitudinal and transverse vibrations in a bar (Table 13.1), show that the lowest longitudinal and transverse modes will have the same frequency when $L = (9/4)\pi K$. Find the ratio of length to diameter for a bar (rod) of circular cross section having the frequency for its lowest transverse and longitudinal vibrations.
6. Determine the frequencies of the (41), (51), and (61) modes of the bell in Fig. 13.32. Show that they are nearly in the ratios 2 : 3 : 4, and that they will produce a strike note (virtual pitch) corresponding to D_5 (see Section 7.4). Is there a vibrational mode with this frequency?
7. In large bells, the (61), (71), (81), and (91) modes create a secondary strike note. Determine the frequencies of these modes in Fig. 13.31, and estimate their virtual pitch (see Section 7.4). What note on the scale is this nearest (see Table 9.2)?
8. If a bell does not have perfect circular symmetry (perfection is seldom achieved in practice), one or more modes of vibration will show "doublet" behavior: that is, there will be two modes of vibration with slightly different frequencies. Suppose that such a doublet exists, having components with frequencies of 440 and 442 Hz. At what rate will the bell "warble"? How is it possible to minimize this warble?

EXPERIMENTS FOR HOME, LABORATORY, AND CLASSROOM DEMONSTRATION

Home and Classroom Demonstration

1. *Modes of a rectangular bar* Sound the first three modes of a steel or aluminum bar (Fig. 13.2 suggests where to hold it and where to strike it for each mode). Note the musical intervals between the modes (the ratio $f_2/f_1 = 2.56$, for example, is slightly less than an octave plus a fourth). Repeat with a bar of hardwood (preferably rosewood) and note the difference.

2. *Modes of a tuned bar* Sound the first three modes of a tuned marimba, vibraphone, and/or xylophone bar from the lower part of the scale. Note the musical intervals between the modes.

3. *Vibraphone resonators* Demonstrate the difference in loudness and timbre of a vibraphone (or vibraharp) when the resonators are open and closed. Demonstrate the intensity vibrato obtained by rotating the resonator discs at different rates.

4. *Chimes* Locate and mark the points where holding a chime firmly will preferentially excite vibrational modes 2 through 6. (They are approximately at $0.5L$, $0.36L$, $0.28L$, $0.22L$, and $0.17L$, where L is its length.) Then show that modes 4, 5, and 6 are nearly in a harmonic 2 : 3 : 4 frequency ratio, with the audible strike note being one octave below mode 4 and two octaves below mode 6.

5. *Kettledrum* Contrast the difference in the tone obtained by striking a kettledrum at its center (which emphasizes the symmetric modes) and at the normal playing spot (about one-fourth of the way from edge to center); see Fig. 13.11.

6. *Kettledrum* Stretch your fingers as far as possible to touch points on the nodal lines of the $(2, 1)$ mode (see Fig. 13.9). Show that the $(2, 1)$ mode is a fifth above the fundamental $(1, 1)$ mode.

7. *Kettledrum* Show how the pedal, by changing the tension, raises the frequency of all modes nearly proportionately and maintains the tuning of the overtones quite well (but not exactly).

8. *Cymbals* Strike a cymbal with a wooden drum stick to show how the timbre (especially the way the "aftersound" develops) varies with strike point and the strength of the blow.